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Compatibility of a Closed Singular Universe with Present Observational Data

A. G. Agnese and M. La Camera

Istituto di Scienze Fisiche dell'Università — Genova, Italy and Istituto Nazionale di Fisica Nucleare — Sezione di Genova, Italy

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The influence of the constant term Λ on the cosmological equations is investigated. It is shown that a closed singular universe is possible even with the presently known matter and radiation densities, provided that the universe age is restricted to a suitable range.

One of the purposes of observational cosmology is to restrict the range of the possible models of the universe and even to select a single model by comparing the predictions with observations.

Nowadays experimental evidence seems to favor singular models, the simplest and most popular of which is the zero-pressure, zero-cosmological-constant, Friedmann model. It is well known, however, that the assumption $\Lambda = 0$ is practically equivalent to rejecting all closed types of universes, unless one assumes that a large amount of matter and/or radiation is missing from present measurements. Leaving out aesthetic or philosophical consideration, we therefore prefer to retain the constant term Λ in cosmological equations, and, making use of the available observational data, we intend to show that this general case is compatible with an isotropic and homogeneous closed singular universe containing both matter and radiation (cf. Agnese, et al., 1970; Kubo, 1970; Fukui, 1973, 1975; Zimmerman and Hellings, 1980).

Of course this conclusion will not appear quite original, but it is a fact that current literature on the subject seems to forget it.

Moreover, if the universe has to be closed, we shall obtain some nontrivial numerical relations between observables of cosmological interest.

We should also like to stress that the occurrence of the cosmological constant in Einstein's equations should not be seen as a mere mathematical

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event: in fact, following Zeldovich's point of view, the cosmological constant, interpreted as proportional to the density and pressure of vacuum fluctuations, could provide a deep physical link between gravitation and elementary particle theories (Zeldovich, 1968; Dolgov and Zeldovich, 1981).

Turning to our purposes, we represent the universe models as points in the three-dimensional parameter space (q, σ_r, σ_m). The dimensionless density parameters are given by

$$\sigma_r = \frac{4\pi G\rho_r}{3H^2}, \qquad \sigma_m = \frac{4\pi G\rho_m}{3H^2} \tag{1}$$

and the Hubble and deceleration parameters are defined as

$$H = \frac{\dot{R}}{R}, \qquad q = -\frac{\ddot{R}}{RH^2} \tag{2}$$

where $R(t)$ is the spatial scale factor of the Robertson–Walker line element.

From Einstein's equations, we have for the signature k and the cosmological constant Λ expressed in terms of present epoch values:

$$\frac{Kc^2}{R_0^2 H_0^2} = 4\sigma_{r0} + 3\sigma_{m0} - q_0 - 1 \tag{3}$$

$$\frac{\Lambda}{3H_0^2} = 2\sigma_{r0} + \sigma_{m0} - q_0 \tag{4}$$

If we want to consider only the "visible" mass and the 2.7-K radiation, we have the following restrictions on the density parameters:

$$\sigma_{r0} = 1.0 \times 10^{-4}$$
$$0.02 \leqslant \sigma_{m0} \leqslant 0.20 \tag{5}$$

Of course the radiation parameter actually makes a negligible contribution, its influence being dominating only at very early epochs.

As to the deceleration parameter, we could in principle use the Hubble brightness–red-shift diagram.

The method suffers several drawbacks, however.

In the first place, as can be easily checked by numerical computations, it is very sensitive to the allowed set of parameters $(q_0, \sigma_{r0}, \sigma_{m0})$ in the measured range of red shifts. Moreover, the corrections due to evolutionary effects, which are not well known, make dubious even the determination of the sign of the deceleration parameter q_0.

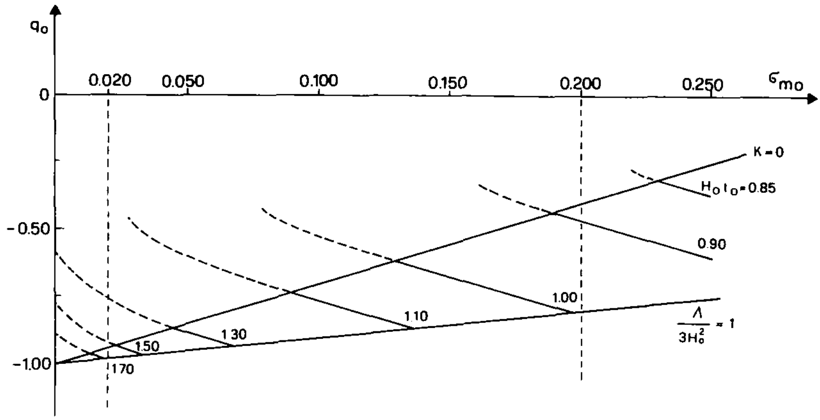

Fig. 1. The age of the universe for different values of q_0 and σ_{m0}, at fixed $\sigma_{r0} = 1.0 \times 10^{-4}$. The line $K = 0$ separates open models (above) from closed models (below).

For a more detailed discussion on these questions, together with a recently proposed method of q_0 measurement, one can see the works of Tammann et al. (1980) and Tinsley (1980).

It seems therefore more convenient to look for another observable quantity which depends on the set (q, σ_r, σ_m), and this will be the universe age, which can be evaluated by means of the integral

$$H_0 t_0 = \int_0^1 \frac{x\,dx}{\left[(2\sigma_{r0} + \sigma_{m0} - q_0)x^4 - (4\sigma_{r0} + 3\sigma_{m0} - q_0 - 1)x^2 + 2\sigma_{m0}x + 2\sigma_{r0}\right]^{1/2}} \tag{6}$$

where $x = R(t)/R(t_0)$. The numerical calculations of the universe age (6) are summarized in Figure 1, where is represented a section of the three-dimensional surfaces $H_0 t_0 = \text{const}$ in the parameter space with the plane $\sigma_{r0} = 1.0 \times 10^{-4}$.

In the same figure, by using equations (3), (4), and (5) are drawn the lines which, in the case of a closed monotonic world of the first kind, give the following *a priori* limitations on the deceleration parameter:

$$-0.98 < q_0 < -0.40 \tag{7}$$

Correspondingly we have for the cosmological constant

$$1 > \frac{\Lambda}{3H_0^2} > 0.60 \tag{8}$$

We can then see from Figure 1 that if the universe age is such that $0.9 \leqslant H_0 t_0 \leqslant 1.5$, the aforeconsidered model is compatible with observational data.

Taking Sandage's determination of the Hubble constant $H_0 = 50$ km s^{-1} Mpc^{-1}, this age restriction corresponds to a universe age comprised between 17 and 29 billion years. A lower bound on $H_0 t_0$ may come from the ages of globular clusters, which are estimated to be between 8 and 18 billion years (Iben, 1974). This ensures that the minimum age restriction is fulfilled, since the universe must have an age greater than the oldest globular clusters.

As to the maximum age restriction, its fulfillment is clearly not so simple to be verified. However, to our present knowledge, the ages of the oldest objects in the universe lie below this limit.

Therefore a closed singular universe is compatible with present observational data.

The type of universe we are considering will now be in a stage dominated by vacuum fluctuations, whose corresponding density $\rho_\Lambda = \Lambda / 8\pi G$ has definitively overwhelmed both matter and radiation densities.

This model is evolving toward the point $(-1,0,0)$ of the parameter space and so its metric will approach the form

$$ds^2 = c^2\,dt^2 - \exp\left[2\left(\frac{\Lambda}{3}\right)^{1/2}(t - t_0)\right]\frac{\left[dr^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right]}{\left(1 + Kr^2/4R_0^2\right)^2} \tag{9}$$

If we want an insight into the past history of this closed universe, we can calculate, for example, its age when matter began to predominate over radiation and when matter was overcome by vacuum fluctuations.

The results are strongly dependent on the actual set of parameters we choose, but we found that for q_0 within the range $[-0.41, -0.97]$, and so with present age of the universe lying between 17 and 29 billion years, the first crossing (from radiation to matter era) occurred somewhere between 0.13 and 13 million years after the big bang, and the second crossing (from matter to vacuum fluctuations era) occurred somewhere between 2.6 and 19 billion years ago. As a concluding remark, we notice that the effect of the cosmological term Λ on the expansion rate should be carefully considered in any evolutionary theory of the universe, its influence being quite important, e.g., with respect to some cosmological implications of grand unified theories.

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